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\[ U_i = \frac{q_{in}}{T_i - T_j} + \left( \frac{1}{h_{p,c} + \frac{1}{k_p}} + \frac{1}{h_{c,op}} \right)^{-1} \]  

(6.4.14)

The evaluation of the radiation heat transfer coefficients in Equation 6.4.14 must take into account that the cover is partially transparent. The net radiation between the opaque plate and the partially transparent cover is given by

\[ q = \frac{\alpha_{p,c}(T_p^4 - T_a^4)}{1 - \rho \rho_v} \]  

(6.4.15)

The radiation heat transfer coefficient between the plate and cover is just the net heat transfer divided by the temperature difference:

\[ h_{p,c} = \frac{1}{1 - \rho \rho_v} \]  

(6.4.16)

Whillier (1977) presents top loss coefficients for collector cover systems of one glass cover over one plastic cover, two plastic covers, and one glass cover over two plastic covers.

## 6.5 TEMPERATURE DISTRIBUTION BETWEEN TUBES AND THE COLLECTOR EFFICIENCY FACTOR

The temperature distribution between two tubes can be derived if we temporarily assume the temperature gradient in the flow direction is negligible. Consider the sheet-tube configuration shown in Figure 6.5.1. The distance between the tubes is \( W \), the tube diameter is \( D \), and the sheet is thin with a thickness \( \delta \). Because the sheet material is a good conductor, the temperature gradient through the sheet is negligible. We will assume the sheet above the bond is at some local base temperature \( T_b \). The region between the centerline separating the tubes and the tube base can then be considered as a classical fin problem.

The fin, shown in Figure 6.5.2(a), is of length \( (W-D)/2 \). An elemental region of width \( \Delta x \) and unit length in the flow direction is shown in Figure 6.5.2(b). An energy balance on this element yields

\[ S \Delta x - U_L \Delta x(T - T_b) + \left( -k \frac{dT}{dx} \right) \bigg|_{x} - \left( -k \frac{dT}{dx} \right) \bigg|_{x + \Delta x} = 0 \]  

(6.5.1)

where \( S \) is the absorbed solar energy defined by Equation 5.9.1. Dividing through by \( \Delta x \) and finding the limit as \( \Delta x \) approaches zero yield

\[ \frac{d^2T}{dx^2} = \frac{U_L}{k} \left( T - T_s - \frac{S}{U_L} \right) \]  

(6.5.2)

The two boundary conditions necessary to solve this second-order differential equation are symmetry at the centerline and the known base temperature:

\[ \left. \frac{dT}{dx} \right|_{x=0} = 0, \quad T_{x=(W-D)/2} = T_s \]  

(6.5.3)

For convenience, we can define two variables, \( m \) and \( \Psi \):

\[ m = \frac{\sqrt{U_L}}{\sqrt{k}} \]  

(6.5.4a)

\[ \psi = T - T_s - \frac{S}{U_L} \]  

(6.5.4b)

and Equation 6.5.2 becomes

\[ \frac{d^2\psi}{dx^2} - m^2 \psi = 0 \]  

(6.5.5)

which has the boundary conditions

\[ \left. \frac{d\psi}{dx} \right|_{x=0} = 0, \quad \psi_{x=(W-D)/2} = T_s - T_s - \frac{S}{U_L} \]  

(6.5.6)

The general solution is

\[ \psi = C_1 \sinh mx + C_2 \cosh mx \]  

(6.5.7)

The constants \( C_1 \) and \( C_2 \) can be found by substituting the boundary conditions into the general solution. The result is
Flat-Plate Collectors

A solar collector is a special kind of heat exchanger that transforms solar radiant energy into heat. A solar collector differs in several respects from more conventional heat exchangers. The latter usually accomplish a fluid-to-fluid exchange with high heat transfer rates and with radiation as an unimportant factor. In the solar collector, energy transfer is from a distant source of radiant energy to a fluid. The flux of incident radiation is, at best, approximately 1100 W/m² (without optical concentration), and it is variable. The wavelength range is from 0.3 to 3 μm, which is considerably shorter than that of the emitted radiation from most energy-absorbing surfaces. Thus, the analysis of solar collectors presents unique problems of low and variable energy fluxes and the relatively large importance of radiation.

Flat-plate collectors can be designed for applications requiring energy delivery at moderate temperatures, up to perhaps 100°C above ambient temperature. They use both beam and diffuse solar radiation, do not require tracking of the sun, and require little maintenance. They are mechanically simpler than concentrating collectors. The major applications of these units are in solar water heating, building heating, air conditioning, and industrial process heat. Passively heated buildings can be viewed as special cases of flat-plate collectors with the room or storage wall as the absorber. Passive systems are discussed in Chapter 14.

The importance of flat-plate collectors in thermal processes is such that their thermal performance is treated in considerable detail. This is done to develop an understanding of how the component functions. In many practical cases of design calculations, the equations for collector performance are reduced to relatively simple forms. The last sections of this chapter treat testing of collectors, the use of test data, and some practical aspects of manufacture and use of these heat exchangers. Costs will be considered in chapters on applications.

6.1 DESCRIPTION OF FLAT-PLATE COLLECTORS

The important parts of a typical liquid heating flat-plate solar collector, as shown in Figure 6.1.1, are the "black" solar energy-absorbing surface with means for transferring the absorbed energy to a fluid, envelopes transparent to solar radiation over the absorber surface that reduce convection and radiation losses to the atmosphere, and heat insulation to reduce conduction losses. Figure 6.1.1 depicts a water heater, and most of the analysis of this chapter is concerned with this geometry. Air heaters are fundamentally the same except that the fluid tubes are replaced by ducts. Flat-plate collectors are almost always mounted in a stationary position (e.g., as an integral part of a wall or roof structure) with an orientation optimized for the particular location in question for the time of year in which the solar device is intended to operate.

6.2 BASIC FLAT-PLATE ENERGY BALANCE EQUATION

In steady state, the performance of a solar collector is described by an energy balance that indicates the distribution of incident solar energy into useful energy gain, thermal losses, and optical losses. The solar radiation absorbed by a collector per unit area of absorber S is equal to the difference between the incident solar radiation and the optical losses as defined by Equation 5.9.1. The thermal energy lost from the collector to the surroundings by conduction, convection, and infrared radiation can be represented as the product of a heat transfer coefficient U times the difference between the mean absorber plate temperature Tpm and the ambient temperature Ta. In steady state the useful energy output of a collector of area A is the difference between the absorbed solar radiation and the thermal loss:

\[
Q_u = A \left( S - U_c(T_{pm} - T_a) \right)
\] (6.2.1)

The problem with this equation is that the mean absorber plate temperature is difficult to calculate or measure since it is a function of the collector design, the incident solar radiation, and the entering fluid conditions. Part of this chapter is devoted to reformulating Equation 6.2.1 so that the useful energy gain can be expressed in terms of the inlet fluid temperature and a parameter called the collector heat removal factor, which can be evaluated analytically from basic principles or measured experimentally.

Equation 6.2.1 is an energy rate equation and, in SI units, yields the useful energy gain per unit time:

\[
\text{In watts (W)}
\]

\[
\text{when } S \text{ is expressed in W/m}^2 \text{ and } U_c \text{ in W/m}^2 \text{ K.}
\]

The most convenient time base for solar radiation is hours rather than seconds since this is the normal period for reporting of meteorological data. (For example, Table 2.5.2 gives solar radiation in W/m² for 1-h time periods.) This is the time basis for S in Equation 5.9.1 since the meaning of 1 is hourly 1/m². We can consider S to be an average energy rate over a 1-h period with units of W/m² h, in which case the thermal loss term \( U_c(T_{pm} - T_a) \) must be multiplied by 3600 s/h to obtain numerical values of the useful energy gain in J/h.
The hour time base is not a proper use of SI units, but this interpretation is often convenient. Alternatively, we can integrate Equation 6.2.1 over a 1-h period. Since we seldom have data over time periods less than 1 h, this integration can be performed only by assuming that $S$, $T_{pm}$, and $T_a$ remain constant over the hour. The resulting form of Equation 6.2.1 is unchanged except that both sides are multiplied by 3600 s/h. To avoid including this constant in expressions for useful energy gain on an hourly basis, we could have used different symbols for rates and for hourly integrated quantities (e.g., $Q$ and $Q_j$). However, the intended meaning is always clear from the use of either $G$ or $J$ in the evaluation of $S$, and we have found it unnecessary to use different symbols for collector useful energy gain on an instantaneous basis or an hourly integrated basis. From a calculation standpoint the 3600 must still be included since $S$ will be known for a hour-time period but the loss coefficient will be in SI units.

A measure of collector performance is the collection efficiency, defined as the ratio of the useful gain over some specified time period to the incident solar energy over the same time period:

$$
\eta = \frac{\int Q_j \, dt}{A \int G \, dt}
$$

(6.2.2)

The design of a solar energy system is concerned with obtaining minimum-cost energy. Thus, it may be desirable to design a collector with an efficiency lower than is technologically possible if the cost is significantly reduced. In any event, it is necessary to be able to predict the performance of a collector, and that is the basic aim of this chapter.

6.3 TEMPERATURE DISTRIBUTIONS IN FLAT-PLATE COLLECTORS

The detailed analysis of a solar collector is a complicated problem. Fortunately, a relatively simple analysis will yield very useful results. These results show the important variables, how they are related, and how they affect the performance of a solar collector. To illustrate these basic principles, a liquid heating collector, as shown in Figure 6.3.1, will be examined first. The analysis presented follows the basic derivation by Whillier (1953, 1977) and Hottel and Whillier (1958).

To appreciate the development that follows, it is desirable to have an understanding of the temperature distribution that exists in a solar collector constructed as shown in Figure 6.3.1. Figure 6.3.2(a) shows a region between two tubes. Some of the solar energy absorbed by the plate must be conducted along the plate to the region of the tubes. Thus the temperature midway between the tubes will be higher than the temperature in the vicinity of the tubes. The temperature above the tubes will be nearly uniform because of the presence of the tube and weld metal.

The energy transferred to the fluid will heat the fluid, causing a temperature gradient to exist in the direction of flow. Since in any region of the collector the general temperature level is governed by the local temperature level of the fluid, a situation as shown in Figure 6.3.2(b) is expected. At any location $y$, the general temperature distribution in the $x$ direction is as shown in Figure 6.3.2(c), and at any location $x$, the temperature distribution in the $y$ direction will look like Figure 6.3.2(d).

To model the situation shown in Figure 6.3.2, a number of simplifying assumptions can be made to lay the foundations without obscuring the basic physical situation. These assumptions are as follows:

1. Performance is steady state.
2. Construction is of sheet and parallel tube type.
3. The headers cover a small area of collector and can be neglected.
4. The headers provide uniform flow to tubes.
5. There is no absorption of solar energy by a cover insofar as it affects losses from the collector.
6. Heat flow through a cover is one dimensional.
7. There is a negligible temperature drop through a cover.
8. The covers are opaque to infrared radiation.
9. There is one-dimensional heat flow through back insulation.
10. The sky can be considered as a blackbody for long-wavelength radiation at an equivalent sky temperature.
11. Temperature gradients around tubes can be neglected.
12. The temperature gradients in the direction of flow and between the tubes can be treated independently.
13. Properties are independent of temperature.
14. Loss through front and back are to the same ambient temperature.
15. Dust and dirt on the collector are negligible.
16. Shading of the collector absorber plate is negligible.

In later sections of this chapter many of these assumptions will be relaxed.

6.4 COLLECTOR OVERALL HEAT LOSS COEFFICIENT

The equations developed in the remainder of this text are often coupled nonlinear algebraic and/or differential equations. The equations are presented in a manner that is convenient for solving by hand or by programming in structured languages such as FORTRAN, Pascal, or C. Typically this means nonlinear equations are linearized, differential equations are discretized, and iterative solutions are required. A number of computer programs are available that can solve systems of algebraic and differential equations; it is only necessary to write the equations in a natural form and let the program organize the solution. The authors use Engineering Equation Solver (EES) to check solutions to the example problems, to solve the homework problems in Appendix A, and to carry on research with their colleagues and graduate students.

It is useful to develop the concept of an overall loss coefficient for a solar collector to simplify the mathematics. Consider the thermal network for a two-cover system shown in Figure 6.4.1. At some typical location on the plate where the temperature is $T_p$, solar energy of amount $S$ is absorbed by the plate, where $S$ is equal to the incident solar radiation reduced by optical losses as shown in Section 5.9. This absorbed energy $S$ is distributed to thermal losses through the top and bottom and to useful energy gain. The purpose of this section is to convert the thermal network of Figure 6.4.1 to the thermal network of Figure 6.4.2.

The energy loss through the top is the result of convection and radiation between parallel plates. The steady-state energy transfer between the plate at $T_p$ and the first cover at $T_{c1}$ is the same as between any other two adjacent covers and is also equal to the energy lost to the surroundings from the top cover. The loss through the top per-unit area is then equal to the heat transfer from the absorber plate to the first cover.

---

1Engineering Equation Solver information is available at www.fchart.com.
The energy conducted to the region of the tube per unit of length in the flow direction can now be found by evaluating Fourier's law at the fin base:

\[ q_{in} = -k \frac{dT}{dx} \left|_{x = (W - D)/2} \right. = (k \delta m/U_L)[S - U_L(T_b - T_f)] \tanh \left( \frac{m(W - D)}{2} \right) \]  

(6.5.9)

but \( k \delta m/U_L \) is just \( 1/m \). Equation 6.5.9 accounts for the energy collected on only one side of a tube; for both sides, the energy collection is

\[ q_{in} = (W - D)[S - U_L(T_b - T_f)] \tanh \left( \frac{m(W - D)/2}{2} \right) \]  

(6.5.10)

It is convenient to use the concept of a fin efficiency to rewrite Equation 6.5.10 as

\[ q_{in} = -(W - D)F[S - U_L(T_b - T_f)] \]  

(6.5.11)

where

\[ F = \frac{\tanh[m(W - D)/2]}{m(W - D)/2} \]  

(6.5.12)

The function \( F \) is the standard fin efficiency for straight fins with rectangular profile and is plotted in Figure 6.5.3.

The useful gain of the collector also includes the energy collected above the tube region. The energy gain for this region is

\[ q_{cone} = D[S - U_L(T_b - T_f)] \]  

(6.5.13)

and the useful gain for the tube and fin per unit of length in the flow direction is the sum of Equations 6.5.11 and 6.5.13:

\[ q' = (W - D)F[S - U_L(T_b - T_f)] \]  

(6.5.14)

Ultimately, the useful gain from Equation 6.5.14 must be transferred to the fluid. The resistance to heat flow to the fluid results from the bond and the tube-to-fluid resistance. The useful gain can be expressed in terms of the two resistances as

\[ q' = \frac{T_b - T_f}{\frac{1}{h_f m D} + \frac{1}{C_b}} \]  

(6.5.15)

where \( D \) is the inside tube diameter and \( h_f \) is the heat transfer coefficient between the fluid and the tube wall. The bond conductance \( C_b \) can be estimated from knowledge of the bond thermal conductivity \( k_b \), the average bond thickness \( \gamma \), and the bond width \( h_b \).

On a per-unit-length basis,

\[ C_b = \frac{k_b h_f}{\gamma} \]  

(6.5.16)

The bond conductance can be very important in accurately describing collector performance. Whiller and Saluja (1965) have shown by experiments that simple wiring or clamping of the tubes to the sheet results in low bond conductance and significant loss of performance. They conclude that it is necessary to have good metal-to-metal contact so that the bond conductance is greater than 30 W/m°C.

We now wish to eliminate \( T_b \) from the equations and obtain an expression for the useful gain in terms of known dimensions, physical parameters, and the local fluid temperature. Solving Equation 6.5.15 for \( T_b \), substituting it into Equation 6.5.14, and solving the result for the useful gain, we obtain

\[ q' = W F'[S - U_L(T_f - T_f)] \]  

(6.5.17)

where the collector efficiency factor \( F' \) is given as

\[ F' = \frac{1}{1/U_{L}} \]  

(6.5.18)

A physical interpretation for \( F' \) results from examining Equation 6.5.17. At a particular location, \( F' \) represents the ratio of the actual useful energy gain to the useful gain that would result if the collector absorbing surface had been at the local fluid temperature.
6.6 Temperature Distribution in Flow Direction

The useful gain per unit flow length as calculated from Equation 6.5.17 is ultimately transferred to the fluid. The fluid enters the collector at temperature \( T_{f,i} \) and increases in temperature until at the exit it is \( T_{f,o} \). Referring to Figure 6.6.1, we can express an energy balance on the fluid flowing through a single tube of length \( \Delta y \) as

\[
\left( \frac{m}{n} \right) C_{p} T_{f,i} - \left( \frac{m}{n} \right) C_{p} T_{f,i} + \Delta y q'_{w} = 0
\]  

(6.6.1)

where \( m \) is the total collector flow rate and \( n \) is the number of parallel tubes. Dividing through by \( \Delta y \), finding the limit as \( \Delta y \) approaches zero, and substituting Equation 6.5.17 for \( q'_{w} \), we obtain
where \( h_{cp-e} \) is the convection heat transfer coefficient between two inclined parallel plates from Chapter 3. If the definition of the radiation heat transfer coefficient (Equation 3.10.1) is used, the heat loss becomes

\[
q_{loss, cp} = (h_{cp-e} + h_{cp-c})(T_p - T_c)
\]

(6.4.2)

The resistance \( R_3 \) can then be expressed as

\[
R_3 = \frac{1}{h_{cp-e} + h_{cp-c}}
\]

(6.4.4)

A similar expression can be written for \( R_i \), the resistance between the covers. In general, we can have as many covers as desired, but the practical limit is two and most collectors use one.

The resistance from the top cover to the surroundings has the same form as Equation 6.4.4, but the convection heat transfer coefficient \( h_c \) is given in Section 3.15. The radiation resistance from the top cover accounts for radiation exchange with the sky at \( T_s \). For convenience, we reference this resistance to the ambient temperature \( T_o \), so that the radiation heat transfer coefficient can be written as

\[
h_{c-e-a} = \frac{\varepsilon(T_s + T_o)(T_p + T_o)}{T_p - T_o}
\]

(6.4.5)

The resistance to the surroundings \( R_i \) is then given by

\[
R_i = \frac{1}{h_c + h_{c-e-a}}
\]

(6.4.6)

For this two-cover system, the top loss coefficient from the collector plate to the ambient is

\[
U_i = \frac{1}{R_1 + R_2 + R_3}
\]

(6.4.7)

The procedure for solving for the top loss coefficient using Equations 6.4.1 through 6.4.7 is necessarily an iterative process. First a guess is made of the unknown cover temperatures, from which the convective and radiative heat transfer coefficients between parallel surfaces are calculated. With these estimates, Equation 6.4.7 can be solved for the top loss coefficient. The top heat loss is the top loss coefficient times the temperature difference, and since the energy exchange between plates must be equal to the overall heat loss, a new set of cover temperatures can be calculated. Beginning at the absorber plate, a new temperature is calculated for the first cover. This new first cover temperature is used to find the next cover temperature, and so on. For any two adjacent covers or plate, the new temperature of plate or cover \( i \) can be expressed in terms of the temperature of plate or cover \( i-1 \) as

\[
T_i = T_{i-1} - \frac{U_i(T_p - T_{i-1})}{h_{i-1} + h_{i-1-i}}
\]

(6.4.8)

The process is repeated until the cover temperatures do not change significantly between successive iterations. The following example illustrates the process.

**Example 6.4.1**

Calculate the top loss coefficient for an absorber with a single glass cover having the following specifications:

- Plate-to-cover spacing: 25 mm
- Plate emittance: 0.95
- Ambient air and sky temperature: 10°C
- Wind heat transfer coefficient: 10 W/m²°C
- Mean-plate temperature: 100°C
- Collector tilt: 45°
- Glass emittance: 0.88

**Solution**

For this single-glass-cover system, Equation 6.4.7 becomes

\[
U_i = \frac{1}{h_{c-e-a} + h_{c-e-a} + h_{c-e-a}}
\]

The convection coefficient between the plate and the cover \( h_{cp-c} \) can be found using the methods of Section 3.11. The radiation coefficient from the plate to the cover \( h_{cp-e} \) is

\[
h_{cp-e} = \frac{\varepsilon(T_p + T_o)(T_p + T_o)}{1 + \varepsilon_p} + \frac{1}{1 + \varepsilon_c}
\]

The radiation coefficient for the cover to the air \( h_{c-e-a} \) is given as

\[
h_{c-e-a} = \varepsilon_o \sigma(T_s + T_o)(T_c + T_o)
\]

The equation for the cover glass temperature is based on Equation 6.4.8:

\[
T_c = T_p - \frac{U_i(T_p - T_o)}{h_{c-e-a} + h_{c-e-a}}
\]

The procedure is to estimate the cover temperature, from which \( h_{c-e-a} \) and \( h_{c-e-a} \) are calculated. With these heat transfer coefficients and \( h_{cp-e} \), the top loss coefficient **
6.7 COLLECTOR HEAT REMOVAL FACTOR AND FLOW FACTOR

It is convenient to define a quantity that relates the actual useful energy gain of a collector to the useful gain if the whole collector surface were at the fluid inlet temperature. This quantity is called the collector heat removal factor \( F_r \). In equation form it is

\[
F_r = \frac{mC_p [T_f - T_i]}{A[U_f U_L]} = \frac{mC_p [S - U_f (T_f - T_i)]}{A[U_f U_L]}
\]

The collector heat removal factor can be expressed as

\[
F_r = \frac{mC_p}{A[U_f U_L]} \left[ \frac{T_f - T_i}{S - U_f (T_f - T_i)} \right]
\]

or

\[
F_r = \frac{mC_p}{A[U_f U_L]} \left[ \frac{T_f - T_i}{S - U_f (T_f - T_i)} \right] = \frac{mC_p}{A[U_f U_L]} \left[ \frac{S - U_f (T_f - T_i)}{S - U_f (T_f - T_i)} \right]
\]

which from Equation 6.6.4 can be expressed as

\[
F_r = \frac{mC_p}{A[U_f U_L]} \left[ 1 - \exp \left( \frac{A[U_f F']}{mC_p} \right) \right]
\]

To present Equation 6.7.4 graphically, it is convenient to define the collector flow factor \( F^* \) as the ratio of \( F_r \) to \( F' \). Thus

\[
F^* = \frac{F_r}{F'} = \frac{mC_p}{A[U_f U_L]} \left[ 1 - \exp \left( \frac{A[U_f F']}{mC_p} \right) \right]
\]

This collector flow factor is a function of the single variable, the dimensionless collector capacitance rate \( mC_p/A[U_f U_L F'] \), and is shown in Figure 6.7.1.

The quantity \( F_r \) is equivalent to the effectiveness of a conventional heat exchanger, which is defined as the ratio of the actual heat transfer to the maximum possible heat transfer. The maximum possible useful energy gain (heat transfer) in a solar collector occurs when the whole collector is at the inlet fluid temperature; heat losses to the surroundings are then at a minimum. The collector heat removal factor times this maximum possible useful energy gain is equal to the actual useful energy gain \( Q_\text{a} \):

\[
Q_\text{a} = A[U_f U_L] [S - U_f (T_f - T_i)]
\]

\( \text{Dunkle and Cooper (1975) have assumed} \) \( U_f \) is a linear function of \( T_f - T_i \)
inlet fluid temperatures in energy balance calculations. This section is concerned with
the estimation of $U_i$ and $F_p$.

The methods for calculating thermal losses from receivers are not as easily sum-
marized as in the case of flat-plate collectors. The shapes and designs are widely variable,
the temperatures are higher, the edge effects are more significant, conduction terms may
be quite high, and the problems may be compounded by nonuniformity of radiation flux
on receivers which can result in substantial temperature gradients across the energy-
absorbing surfaces. It is difficult to present a single general method of estimating thermal
losses, and ultimately each receiver geometry must be analyzed as a special case.

The nature of the thermal losses for receivers of concentrating collectors is the same
as for flat-plate exchangers. Receivers may have covers transparent to solar radiation. If
so, the outward losses from the absorber by convection and radiation to the atmosphere
are correspondingly modified and equations similar to those of Chapter 6 can be used to
estimate their magnitude. As with flat-plate systems, the losses can be estimated as being
independent of the intensity of incident radiation (although this may not be strictly true
if a transparent cover absorbs appreciable solar radiation). In any event, an effective
transmittance-absorptance product can also be defined for focusing systems. Furthermore,
with focusing systems the radiation flux at the receiver is generally such that only cover
materials with very low absorptance for solar radiation can be used without thermal
damage to the cover. Conduction losses occur through the supporting structure and
through insulation on parts of the receiver that are not irradiated.

The generalized thermal analysis of a concentrating collector is similar to that of a
flat-plate collector. Although not necessary, it is convenient to derive appropriate expres-
sions for the collector efficiency factor $F'$, the loss coefficient $U_i$, and the collector heat
removal factor $F_p$. With $F_p$ and $U_i$ known, the collector useful gain can be calculated
from an expression that is similar to that for a flat-plate collector. One significant differ-
ence between a concentrating collector and a flat-plate collector is the high tempera-
tures encountered in the concentrating collector. High temperatures mean that thermal radiation
is important, leading to the loss coefficient being temperature dependent.

As an example of calculating the thermal losses and the loss coefficient $U_i$, consider
an uncovered cylindrical absorbing tube used as a receiver with a linear concentrator. Assume that there are no temperature gradients around the receiver tube. The loss and
loss coefficient considering convection and radiation from the surface and conduction
through the support structure are

$$Q_{loss} = h_u (T_r - T_a) + e_c (T_r - T_{co}) + U_{con}(T_r - T_a)$$

$$= (h_u + h_c + U_{con})(T_r - T_a)$$

$$= U_i (T_r - T_a)$$

(7.3.1)

The linearized radiation coefficient can be calculated from

$$h_r = e_c (T_r - T_{co}) (T_r - T_a)$$

(7.3.2)

If a single value of $U_i$ is not acceptable due to large temperature gradients in the flow
direction, the collector can be considered as divided into segments each with constant
$U_i$, as was done by Stetzel et al. (2004). The estimation of $h_r$ for cylinders is noted in
Section 3.15. Estimation of conductive losses must be based on knowledge of the con-
struction details or on measurements on a particular collector.

Linear concentrators may be fitted with cylindrical absorbers surrounded by trans-
parent tubular covers. For a collector of length $L$ the heat transfer from the receiver at
$T_r$ to the inside of the cover at $T_r'$ through the cover to $T_a$ and then to the surroundings
at $T_a$ and $T_{co}$ is given by

$$Q_{loss} = \frac{2 \pi k_{eff} L}{\ln (D_r/D_c)} (T_r - T_{co})$$

(7.3.3)

$$Q_{loss} = \frac{2 \pi k_{eff} L (T_r - T_{co})}{\ln (D_r/D_c)}$$

(7.3.4)

$$Q_{loss} = \pi D_{co} h_c (T_{co} - T_{co}) + e_c \pi D_{co} L (T_{co} - T_{co})$$

(7.3.5)

where the subscript $r$ represents the receiver and subscripts $ci$ and $co$ represent the cover
inside and outside. The cover thermal conductivity is $k_c$ and $k_{eff}$ is an effective conduc-
tivity for convection between the receiver and the cover and is found from Equation
3.11.5. If the annulus is evacuated so that convection is suppressed, $k_{eff}$ can be zero at
very low pressures. The outside convective coefficient $h_c$ is calculated with Equation
3.15.12.

The procedure used to solve Equations 7.3.3 to 7.3.5 by hand is to estimate $T_r$
(which will be much closer to $T_r'$ than $T_a$), calculate $Q_{loss}$ from Equation 7.3.5, and
substitute these quantities into Equation 7.3.4 to find an estimate of $T_{co}$. Equation 7.3.3
is used to check the initial guess of $T_{co}$ by comparing $Q_{loss}$ from Equation 7.3.5 with
that calculated from Equation 7.3.3.

It may be necessary to account for absorption of solar radiation by the cover. If so, the
absorbed energy can be added to the left-hand side of Equation 7.3.5, which assumes
all of the energy is absorbed on the outside of the cover. The principles are identical
with those shown in Chapter 6 for flat-plate collectors. If there is significant heat loss
through supports to the surroundings, a suitable heat loss equation of the form
$Q_{loss} = (UA)_{loss} \Delta T$ can be included with Equations 7.7.3 to 7.7.5. The total heat loss is then the
sum of $Q_{loss}$ and $Q_{loss}$.

Example 7.3.1

Calculate the loss coefficient $U_i$ for a 60-mm cylindrical receiver at 200°C. The absorber
surface has an emittance of 0.31. The absorber is covered by a glass tubular cover 90
mm in outer diameter and 4 mm thick. The space between the absorber and cover is