Annexure AP20-6

Course Contents for Ph.D.

Comprehensive Exam

Department of Mathematics

Course Title	**Analysis**	Number	MACxx
Department	Mathematics		
Offered for	Comprehensive Examination		

Objectives

1. To evaluate students with basic knowledge of Analysis
2. To examine the knowledge of candidates in the domain of Analysis

Contents

Real analysis: Set theory, finite sets, countable and uncountable sets, sequences, series and their convergence, Heine-Borel theorem, continuity, uniform continuity, differentiability, mean value theorems, metric spaces, completeness, compactness, connectedness, sequence spaces, function spaces, modes of convergence, Riemann sums and Riemann integrals, improper integrals, functions of bounded variation, Lebesgue measure and Lebesgue integral, functions of several variables, directional derivative, partial derivative, derivative as a linear transformation, inverse and implicit function theorems. **[Ref. B1: Chap. 2-6, Ref. R: Chap. 3-5, 7, 11-14]**

Topology: Topological Spaces, basis for a topology, order and product topology, continuous functions, subspace topology, quotient topology, countability axioms, connectedness, compactness, countability axioms, separation axioms, normal spaces, Tychonoff spaces, Uryshon's lemma, Tieze extension theorem, Tychonoff theorem, the Stone-Cech compactification. **[Ref. M: Chap. 2-5]**

Complex analysis: Algebra of complex numbers, polynomials, power series, transcendental functions, Differentiable functions, Analytic functions, Cauchy-Riemann equations, elementary functions, line integral, contour integral, antiderivatives Cauchy-Goursat theorem, Cauchy's integral formula, Liouville's theorem, fundamental theorem of Algebra, maximum modulus principle, Taylor series, Laurent series, residues and poles, Jordan's lemma, Rouche's theorem, conformal mappings, Mobius transformations. **[Ref BC: Chap. 2-9]**

Textbooks/Book Chapters

1. [B1] Bartle, R., and Sherbert, D., (2011), *Introduction to Real Analysis*, 3rd Edition, Wiley.
2. [BC] Brown, J. W. and Churchill, R. V., (2013) *Complex variables and applications*, 9 Ed., McGraw Hill Publications.
3. [M] Munkres J., (2000), *Topology*, 2nd edition, Pearson Publications.
4. [R] Royden, H.L., (1988), *Real Analysis*, 3rd Edition, Macmillan Publishing Company.

Online Course Material

1. Kulkarni, S.H., Real Analysis, NPTEL Course Material, Department of Mathematics, IIT Madras, https://nptel.ac.in/courses/111106053/.
2. Sree Krishna, P.A.S., Complex Analysis, NPTEL Course Material, Department of Mathematics, IIT Guwahati, https://nptel.ac.in/courses/111103070/
3. Veeramani, P., Topology, NPTEL Course Material, Department of Mathematics, IIT Madras, https://nptel.ac.in/courses/111106054/.

Course Title	**Functional Analysis and Measure Theory**	Number	MACxx
Department	Mathematics		
Offered for	Comprehensive Examination		

Objectives

To assess the background of research scholar to carry out research in the broad domain of functional analysis and measure theory

Contents

Functional analysis: Normed linear spaces, Banach spaces, operators on normed linear spaces, algebraic dual and reflexive spaces, linear functionals, inner-product spaces, Hilbert spaces, projection theorem, total orthonormal set, self-adjointness, unitary and normal operators, Riesz representation theorem, Banach contraction mapping theorem, Hahn-Banach extension theorem, open mapping and closed graph theorems, Uniform boundedness Principle, spectral theory of linear operators in normed spaces. **[Ref. LV: Chap. 2-7, EK: Chap, 2-11, RW: Chap. 1-8, 10-12]**

Measure theory: Measure: Algebra, Sigma-algebra, outer measures, measures, Borel measure, and Lebesgue measure. **[Ref. Fo: Chap. 1]**

Integration: Measurable functions, simple functions and their integrals, convergence theorems, modes of convergence, product measure, Fubini-Tonelli's theorem, change of variable formula, integration in polar coordinates, L^p-spaces **[Ref. Fo: Chap. 2]**

Textbooks/Book Chapters

1. [Fo] Folland G. B., (1999) *Real analysis: modern techniques and their applications*, 2nd Edition, Willey.
2. [LV] Limaye, B. V., (1996), *Functional Analysis*, Second Edition, New Age International.
3. [EK] Kreyszig, E., (1989) *Introductory Functional Analysis with Applications*, John Wiley & Sons Inc.
4. [RW] Ruddin, W.,(2017), *Functional Analysis*, 2nd Ed., McGraw Hill Education.

Online Course Material:

1. Measure Theory: Claudio Landim, Masters Program: Measure Theory, IMPA,Rio de Janeiro, Brazil, https://www.youtube.com/playlist?list=PLo4jXE-LdDTQq8ZyA8F8reSQHej3F6RFX.
2. Functional Analysis: Srivastava, P. D., Functional Analysis, NPTEL Course Material, Department of Mathematics, IIT Kharagpur, https://nptel.ac.in/syllabus/111105037/

Course Title	**Statistical Inference and Processes**	Number	MACxx
Department	Mathematics		
Offered for	Comprehensive Examination		

Objectives

To assess the background of research scholar to carry out research in the broad domain of Probability, Statistics and related areas.

Contents

Probability and Distributions: Descriptive statistics, exploratory data analysis, sample space, discrete probability, independent events, Bayes theorem, Random variables and distribution functions (univariate and multivariate), expectation and moments, independent random variables, marginal and conditional distributions, standard discrete and continuous univariate distributions, moment generating function and characteristic function, probability inequalities (Tchebyshef, Markov, Jensen), WLLN, SLLN, central limit theorem (i.i.d. case). **[Ref. R1: Chap. 1-6]**

Statistical Inference: Sampling distributions, standard errors and asymptotic distributions, distribution of order statistics and range, methods of estimation, properties of estimators, confidence intervals, tests of hypotheses: most powerful and uniformly most powerful tests, likelihood ratio tests, analysis of discrete data and chi-square test of goodness of fit, large sample tests, Gauss-Markov models, estimability of parameters, best linear unbiased estimators, confidence intervals, tests for linear hypotheses, analysis of variance and covariance, simple and multiple linear regression, elementary regression diagnostics, logistic regression**[Ref. R1: 7-12, R3: Chap. 5-9,11,12]**

Stochastics Processes Markov chains with finite and countable state space, classification of states, limiting behaviour of n-step transition probabilities, stationary distribution, Poisson and birth-and-death processes. **[Ref. R2: Chap. 4-7, 10]**

Textbooks/Book Chapters

1. [R1] Rohatgi, V. K. and Saleh, A. K. Md. E. (2001), *An Introduction to Probability and Statistics*, 2nd Edition, Wiley.
2. [R2] Ross, S. M. (2005), *Introduction to Probability Models*, Elsevier.
3. [R3] Casella, G. and Berger, R. L. (2003), *Statistical Inference*, Brooks/Cole Publishing Company.

Online Course Material

1. Kumar, S., Probability and Statistics, NPTEL Course Material, Department of Mathematics, IIT Kharagpur, https://nptel.ac.in/courses/111105090/
2. Zhou Fan, Introduction to Statistical Inference, Online Course Material, Stanford University, http://web.stanford.edu/class/stats200/

Course Title	**Theory of Differential Equations**	Number	MACxx
Department	Mathematics		
Offered for	Comprehensive Examination		

Objectives

To assess a candidate's preparedness to carry out research in the broad domain of ordinary and partial differential equations

Contents

ODEs: Existence and uniqueness of solutions of initial value problems for first order ordinary differential equations, singular solutions of first order ODEs, system of first order ODEs, matrix exponential, General theory of homogeneous and non-homogeneous linear ODEs, variation of parameters, variable coefficient second order ODEs, solution by power series, method of Frobenius, special functions, Laplace transform, Sturm-Liouville boundary value problems, Green's function, non-linear ODEs, phase plane method, group theoretical methods, Lie groups, asymptotic methods, stability, instability and bifurcations . [**Ref. K: Chap. 1-13, B: Chap. 1-6, 9-11**]

PDEs: Lagrange and Charpit methods for solving first order PDEs, Cauchy problem for first order PDEs, classification of second order PDEs, general solution of higher order PDEs with constant coefficients, method of separation of variables, similarity solutions, distributions, Fourier transform method, Laplace transform method, Hopf-Cole transformation, Hölder spaces, Sobolev spaces, weak derivatives, traces, Sobolev inequalities, existence of weak solutions, Lax-Milgram theorem, energy estimates, Fredholm alternative, regularity, Max-Min principles, Riemann problem. [**Ref. E: Chap. 2-11, H: Chap, 1-13, S: Chap. 2-14**]

Textbooks/Book Chapters

1. [K] King, A. C., Billingham, J. and Otto, S. R., (2003), *Differential equations: Linear, nonlinear, Ordinary and partial*, Cambridge University Publication.
2. [B] Birkhoff G. and Rota G., (1989), *Ordinary differential equations*, 4th Edition, Wiley.
3. [E] Evans, L. C., (2000), *Partial differential equations*, AMS Publication.
4. [H] Haberman, R., (2019), *Applied partial differential equations*, 5th Edition, Pearson.
5. [S] Strauss, W. A., (2008), *Partial differential equations: An introduction,* 2nd Edition, Wiley.

Online Course Material

1. Miller H. and Mattuck, A.. 18.03 Differential Equations. Spring 2010. Massachusetts Institute of Technology: MIT OpenCourseWare, https://ocw.mit.edu/courses/mathematics/18-03-differential-equations-spring-2010/#
2. Sinha, R. K., Partial differential equations, NPTEL course, Department of Mathematics, IIT Guwahati, https://nptel.ac.in/courses/111/103/111103021/

Course Title	**Linear algebra, numerical analysis and Optimization**	Number	MACxx
Department	Mathematics		
Offered for	Comprehensive Examination		

Objectives

To assess a candidate's preparedness to carry out research in the broad domain of linear algebra, numerical analysis and optimization techniques.

Contents

Linear algebra: Vector spaces, subspaces, linear dependence, basis, dimension, dual spaces, quotient spaces, linear transformations and their algebra, matrices and their algebra, rank and determinant of matrices, linear equations, positive definite matrices, singular value decomposition, minimum principles, eigenvalues and eigenvectors, Cayley-Hamilton theorem, matrix representation of linear transformations, change of basis, projections, adjoints, canonical forms, diagonal forms, triangular forms, Jordan forms, inner product spaces, orthogonality, quadratic forms, reduction and classification of quadratic forms **[Ref. H1: Chap. 2-10, H2: Chap. 1-4]**

Numerical analysis: Numerical solutions of algebraic equations, rate of convergence, direct and iterative methods for solution of systems of linear algebraic equations, relaxation techniques, conjugate gradient method, numerical solution of nonlinear system of equations, numerical methods for computation of eigenvalues and eigenvectors, finite differences, interpolation, numerical differentiation and integration, numerical methods solution of differential equations (ordinary and partial) for initial value problems and boundary value problems, consistency, error analysis **[Ref. B: Chap. 2-12, K: Chap. 2-12]**

Optimization: Linear combination, convex set, convex hull, linear programming problem, feasible solution, basic feasible solution, graphical solution, Simplex method, Charne's M method, two phase method, solving system of equations and finding the inverse of a matrix using simplex method, duality, complementary slackness theorem, duality and simplex method, dual simplex algorithm, integer programming, Gomory's cut-constraint method, Branch and Bound method, transportation problem, assignment problem, first and second order conditions, iterative methods, line search methods, global convergence of descent algorithms, Newton's method, Conjugate direction method, Quasi-Newton Method, constrained optimization - Lagrange Multipliers, Karush-Kuhn-Tucker conditions, regular points, sensitivity analysis, quadratic Programming, convex problems **[Ref. L: Chap. 2-15]**

Textbooks/Book Chapters

1. [H1] Hoffman, K. and Kunze, R, (2010), *Linear algebra*, 2nd Edition, PHI.
2. [H2] Halmos, P. R., (2000), *Finite dimensional vector spaces*, Springer.
3. [B] Burden, R. L. and Faires J. D., (2011), *Numerical analysis*, 9th Edition, Cengage.
4. [K] Kress, R., (1991), *Numerical analysis*, Springer.
5. [L] Luenberger, D. G. and Ye, Y., (2008), *Linear and Nonlinear Programming*, 3rd Edition, Springer.

Online Course Material

1. Gilbert Strang. 18.06 Linear Algebra. Spring 2010. Massachusetts Institute of Technology: MIT OpenCourseWare, https://ocw.mit.edu/courses/mathematics/18-06-linear-algebra-spring-2010/
2. Usha, R., Numerical Analysis, NPTEL Course Material, Department of Mathematics, IIT Madras, https://nptel.ac.in/courses/111106101/
3. Goswami, A., Optimization, NPTEL Course Material, Department of Mathematics, IIT Kharagpur, https://nptel.ac.in/courses/111105039/1

Course Title	**Foundations of Modern Algebra**	Number	MACxx
Department	Mathematics		
Offered for	Comprehensive Examination		

Objectives

To assess a candidate's preparedness to carry out research in the broad domain of Algebra

Contents

Group Theory: Groups, subgroups, cyclic subgroups, normal subgroups and quotient groups, Lagrange's theorem, homomorphisms and isomorphism theorems, Cauchy's theorem, composition series, Jordan-Holder theorem. group action and permutation representation, centralizer and normalizers, stabilizers and kernels, Cayley's theorem and the class equation, automorphisms, inner automorphisms, automorphism groups of some finite groups, simple groups, alternating groups and their simplicity for n > 4, nilpotent and solvable groups, Sylow theorems and applications. **[Ref. D: Chap. 1-6, L: Chap. 1]**

Ring Theory: Rings, polynomial rings, matrix rings, group rings, ring homomorphism, quotient rings and isomorphism theorems, ideals and their properties, prime and maximal ideals, existence of maximal ideals, Chinese remainder theorem, Euclidean domains, PIDs, UFDs, Field of fractions, characteristic, Eisenstein's irreducibility criterion, Gauss' theorem, Non-commutative rings: semi-simple rings and modules, Wedderburn's theorem. **[Ref. D: Chap. 7-9, L: Chap. 2]**

Fields and Galois Theory: Fields, finite fields, fields of characteristic p > 0, algebraic field extension, algebraic closure, perfect field, separable and inseparable extensions, primitive element theorem, extensions of finite fields, cyclotomic polynomials, fundamental theorem of Galois Theory, solvability by radicals, Galois groups of polynomials, Galois groups over rationals, transcendental extensions. **[Ref. D: Chap. 13-14, L: Chap. 6, 11-12]**

Textbooks/Book Chapters

1. [D] Dummit, D. S. and Foote, R. M., (2004), *Abstract algebra*, 3 Ed., Wiley.
2. [L] Lang,S., (2002), *Algebra*, 3rd Edition, Springer.

Online Course Material

1. Hanumanthu, K., *Introduction to abstract group theory,* NPTEL Course Material, Department of Mathematics, Indian Institute of Technology Madras, https://nptel.ac.in/courses/111106113/
2. Hanumanthu, K., *Introduction to rings and fields,* NPTEL Course Material, Department of Mathematics, Indian Institute of Technology Madras, https://nptel.ac.in/courses/111106131/
3. Patil, D. P., *Galois Theory,* NPTEL Course Material, Department of Mathematics, Indian Institute of Technology Bombay, https://nptel.ac.in/courses/111101117/

Course Mapping:

Disciplines	Papers		
Pure Mathematics	Analysis	Foundation of Modern Algebra	Functional analysis and measure theory
Applied Mathematics	Statistical Inference and Processes	Theory of Differential Equations	Linear algebra, numerical analysis and optimization
Data and Computational Sciences			