Title: SIMULTANEOUS CONJUGACY CLASSES AS COMBINATORIAL INVARIANTS OF FINITE GROUPS

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Journal: Communications in algebra

Volume: In Press

Year: Accepted in 2022.

Publisher: Taylor & Francis

Abstract: We consider the problem of counting simultaneous conjugacy classes of n −tuples in a finite group G. Let $\alpha_{\{G,n\}}$ denote the number of simultaneous conjugacy classes of n-tuples, and $\beta_{\{G,n\}}$ the number of simultaneous conjugacy classes of commuting n −tuples in G. We study the generating functions $A_{\{G\}}(t) = \sum_{\{n\geq 0\}} t^n \alpha_{\{G,n\}}$ and $B_{\{G\}}(t) = \sum_{\{n\geq 0\}} t^n \beta_{\{G,n\}}$ which are rational functions of t. We show that $A_G(t)$ determines and is completely determined by the class equation of G. We prove that the normalized functions $A_G\left(\frac{t}{|G|}\right)$ and $B_G\left(\frac{t}{|G|}\right)$ are invariants of isoclinism families. We compute exponential growth factors of $\alpha_{\{G,n\}}$ and $\beta_{\{G,n\}}$.